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An econometric model MLRM + Analytic Hierarchy Process (AHP) to select the best alternative for a real estate investment. Case: Apartment in Panama City, Panama

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Alternative 1

Alternative 2

Alternative 3

Alternative 4

Random sample of offers with total prices

Property	Construction area [m^2]	Bedroom [adim]	Bathroom [adim]	Parking Place [adim]	Age [year]	Balcony [year]	Amenities [adim]	Floor [adim]	Total price [USD]
PH Arenas Park	67	2	1	1	5	Si	Si	23	165.000,00
PH Vista Marina Tower	105	3	2	2	8	Si	Si	16	170.000,00
PH Coral Tower	81	2	2	2	5	Si	Si	26	200.000,00
PH Miradores Residences	90	3	2	2	2	Si	Si	28	168.500,00

Are the prices offered in the real estate portals the "real prices" of the market?



What would be the best real estate investment alternative?

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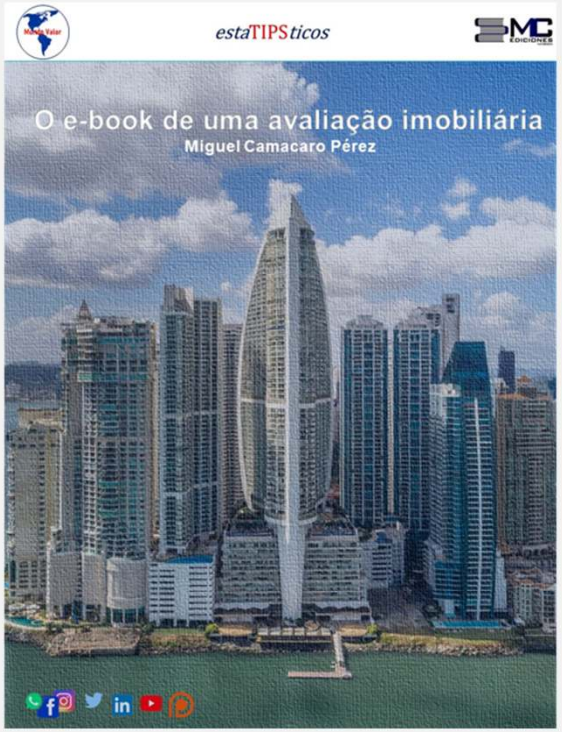
From the Multiple Linear Regression Model (MLRM)

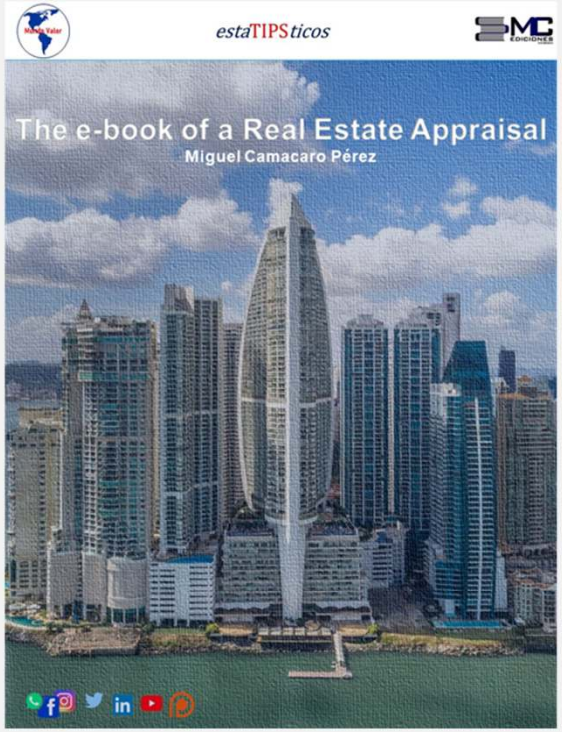
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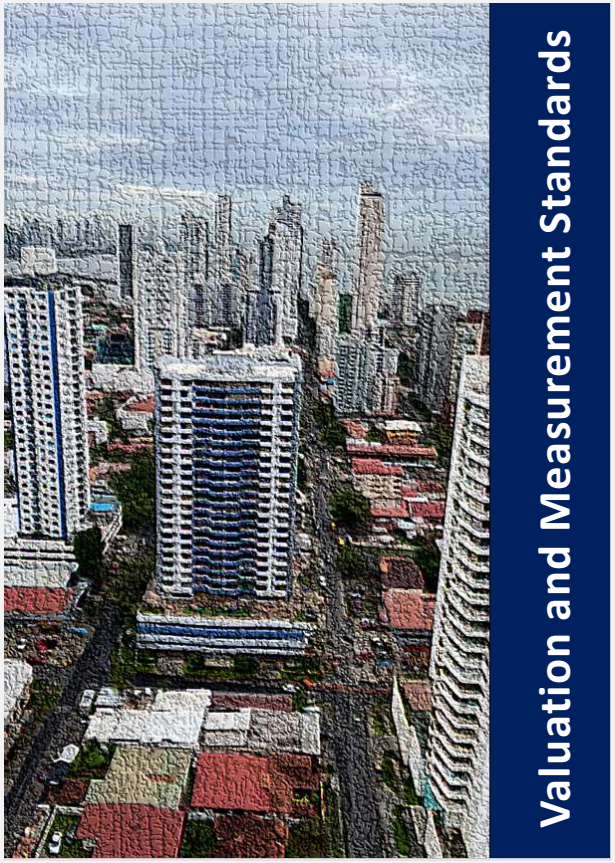




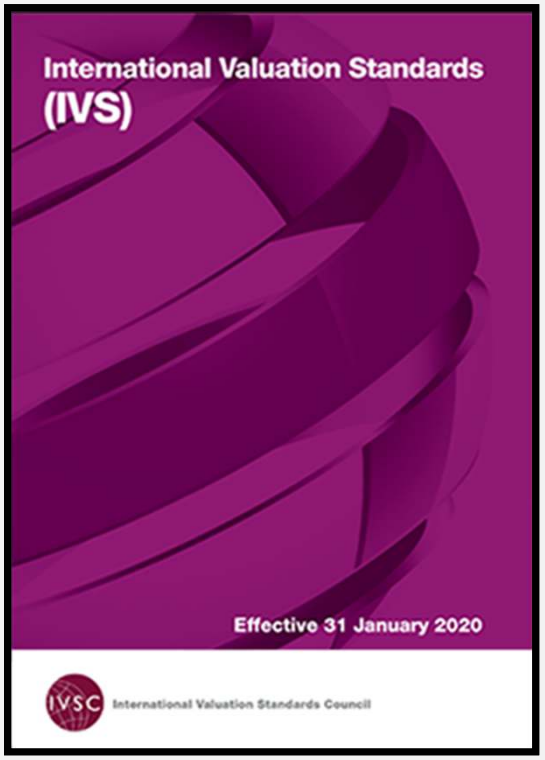
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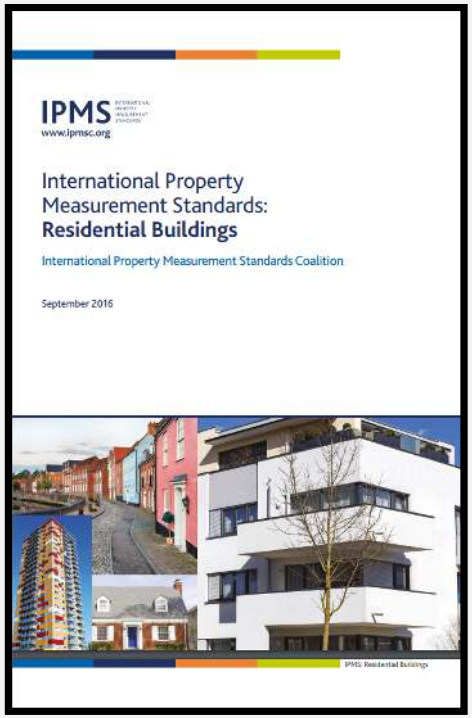


Valuation and Measurement Standards









The direct comparative method of market data and real estate appraisal.

"The estimation of the value of a real estate property follows a research methodology that is based on the statistical analysis of a collected sample of purchase and sale transactions and offers in the real estate market. The results obtained by processing the data (empirical basis) are presented in a technical report identified as a Real Estate Appraisal Report".

Source: Camacaro (2020)

Hedonic pricing method

The value of real estate is estimated based on its intrinsic characteristics (attributes) and extrinsic characteristics (externalities).

V: real estate value

It is based on economic models where the value will be a function of the nature of the real estate and the conditions of the market where it is consumed or traded.

$$V = f(A, B, Pe, C, F, U)$$

An individual will pay more for a property to the extent that its attributes satisfy his needs. The greater the satisfaction, the more he will be willing to consume a product at a higher price.

Variables
(A, B, Pe, C, F, U)

Source: Own elaboration

Linear regression model

"Regression analysis deals with the study of the dependence of the explained variable on one or more variables, the explanatory variables, with the objective of estimating and/or predicting the population mean or average value of the explained variable in terms of known or fixed values (in repeated samples) of the explanatory variables."

Source: Gujarati (1997) - Prepared by the authors.

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

Where:

- Y_i: variable explained
- X_i : explanatory variable
- β_0, β_1: regressors
- ε_i: errors

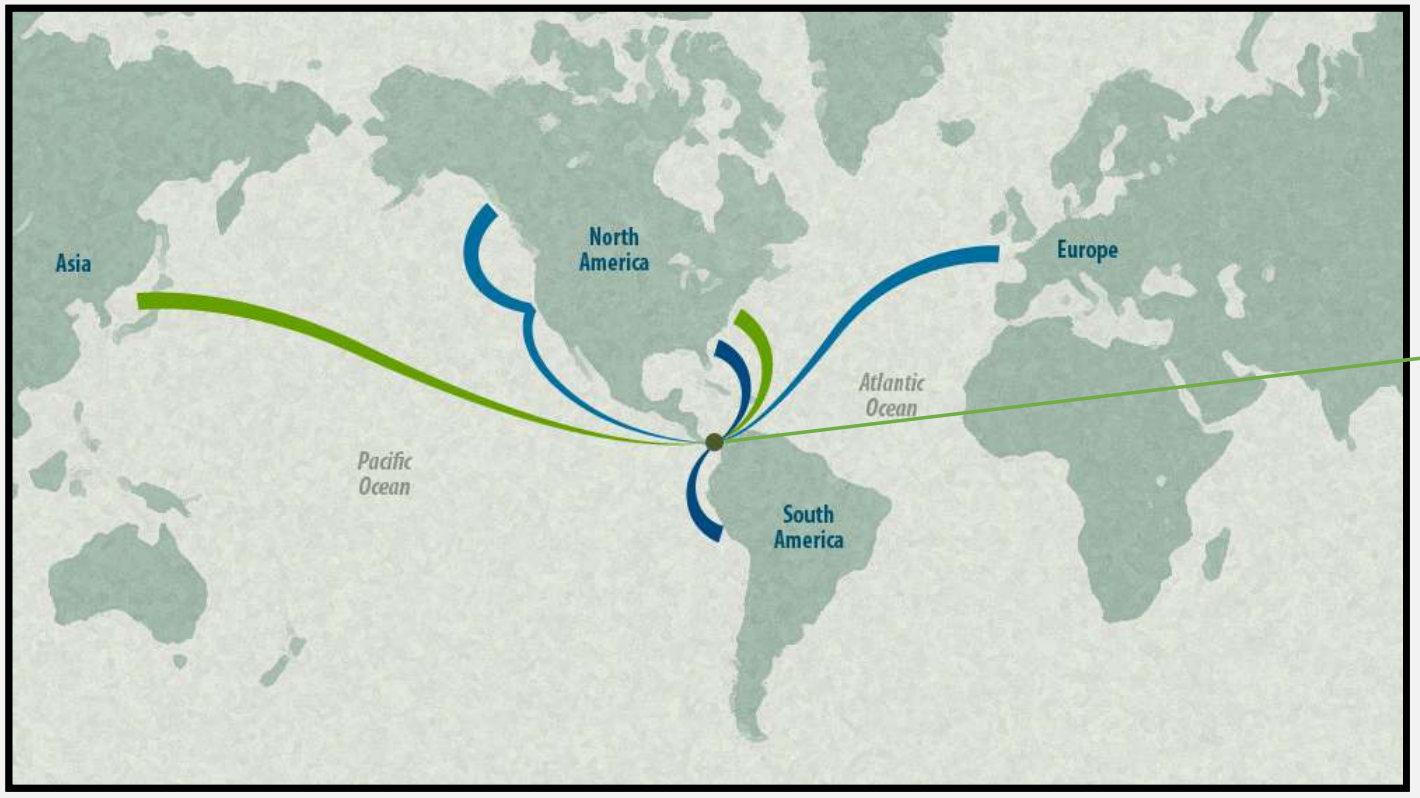
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Price estimation with MLRM

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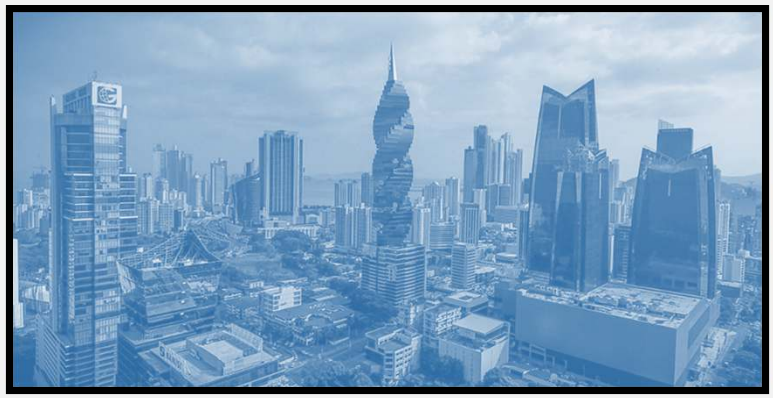
Panamá



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Río Abajo
Vía Simón Bolívar o Transístmica
Parque Recreativo y Cultural Omar
Vía Brasil
Autopista Corredor Sur

Model

Economic

$$PT = f(Ac, Ha, Ba, Pe, Ed, Ca, Ev, Pi, DPV)$$

Econometric

$$PT_i = \beta_0 + \beta_1 Ac_i + \beta_2 Ha_i + \beta_3 Ba_i + \beta_4 Pe_i + \beta_5 Ed_i + \beta_6 Ca_i + \beta_7 Ev_i + \beta_8 Pi_i + \beta_9 DPV_i + \varepsilon_i$$

Population

- The population under study is represented by multi-family residential units, with areas ranging from [60,00 y 140,00] m^2, located in buildings that have between 10 y 40 floors, located in the Corregimiento de Rio Abajo area of Panama City.

Sample

- For the structuring of the Multiple Linear Regression Model (MLRM), a sample with size *n* equal to 97 data corresponding to prices of market offers of multi-family residential units was raised. The physical boundaries of the polygonal analyzed are Vía Simón Bolívar or Transístmica, Río Abajo, Corredor Sur Highway (Pacific Ocean) and Vía Brasil.



Table 5: Variables of the econometric model

Variable	IM	Unit	Description	Variable Type	Measurement Scale
Construction area	*Ac*	m^2	Legal or assessable area of the property.	Explanatory Quantitative Continuous	Reason
Parking place	*Pe*	adim	Number of parking spaces in the property.	Explanatory Quantitative Discrete	Reason
Age	*Ed*	year	Age of the building where the property is located.	Explanatory Quantitative Discrete	Reason
Quality[1]	*C*	adim	High 3 Medium High 2 Medium 1	Explanatory Categorical Discrete	Ordinal
Floors	*Pi*	adim	Number of floors in the building	Explanatory Quantitative Discrete	Reason
Distance to the valorizing pole	*DPV*	m	Distance from the building to the central point of the Omar Recreational and Cultural Park.	Explanatory Proxy Continuous	Reason
Total price of the property	*PT*	UM	Total price of the apartment offers.	Explained Quantitative Continuous	Reason

Own elaboration based on Camacaro & Mock (2021)

$$\widehat{PT} = 78.812,3296.1,0075^{Ac}.1,0428^{Pe}.0,9906^{Ed}.1,0319^{Ca}.1,0063^{Pi}.0,9999^{DPV}$$

Resultados RL

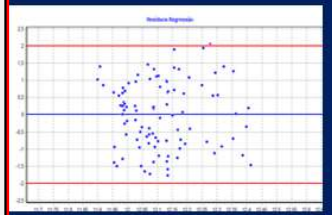
Dados e Variáveis	
Dados	97
Dados Considerados	97
Variáveis	7
Variáveis Consideradas	7
Coeficientes	
Correlação	1 - 0,8592 / 0,8625
Determinação	1 - 0,7382 / 0,7438
R2 Ajustado	1 - 0,7208 / 0,7267
Testes de Hipóteses	
F Calculado	42,3
Significância Modelo	0,01
D Calculado	
Durbin Watson	
Normalidade dos Resíduos	
-1 e +1 desvios padrõe	65%
-1,64 e +1,64 desvios p	93%
-1,96 e +1,96 desvios p	98%
Diversos	
Desvio Padrão	0,07568
Outliers do Modelo	1 (1,03%)
Método de Cálculo	Geral

Variáveis	Transf.	Elasticidade	t Calculado	Sig.(%)	Coef. RL
Ac	x	4,82%	11,90	**0,01%**	0,007470
Pe	x	4,28%	2,21	**2,96%**	0,041880
Ed	x	-1,21%	-2,75	**0,72%**	-0,009395
Ca	x	0,63%	1,78	**7,87%**	0,031409
Pi	x	1,77%	4,25	**0,01%**	0,006284
DPV	x	-1,42%	-3,34	**0,12%**	-0,000071
PT	ln(y)		177,97	**0,01%**	11,287471

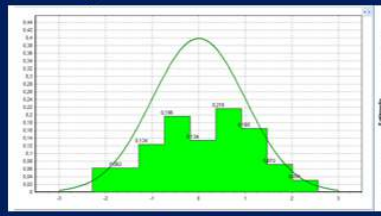
Matriz de correlaciones con influencia

Id	Variável	Transf.	Alias	x1	x2	x3	x4	x5	x6	y
	Ac	x	x1	0	0,01	0,49	0,03	0,23	0,41	0,78
	Pe	x	x2	0,01	0	0,08	0,09	0,10	0,20	0,23
	Ed	x	x3	0,49	0,08	0	0,06	0,34	0,12	0,28
	Ca	x	x4	0,03	0,09	0,06	0	0,05	0,03	0,18
	Pi	x	x5	0,23	0,10	0,34	0,05	0	0,18	0,41
	DPV	x	x6	0,41	0,20	0,12	0,03	0,18	0	0,33
	PT	ln(y)	y	0,78	0,23	0,28	0,18	0,41	0,33	0

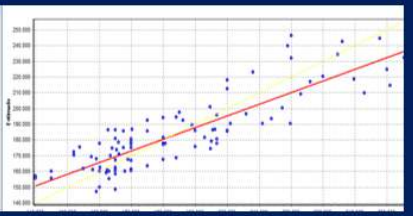
Normalidad – Residuos/regresión



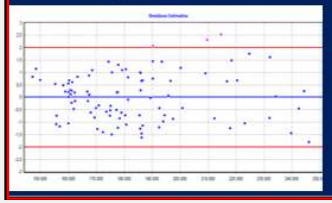
Histograma de frecuencia/errores



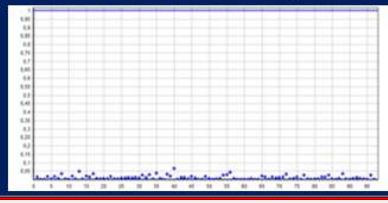
Ajuste del modelo



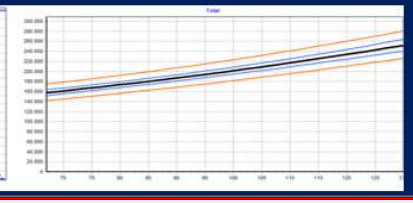
Residuos/estimación - *outlier*



Distancia de *Cook* / Puntos influyentes



Punto mínimo/máximo



Grado de fundamentación

Ítem	Puntos		
1	3	Puntos mínimos	16
2	3		
3	3	Ítems obligatorios	Todos en III
4	3		
5	3		
6	3	Grado de fundamentación	III
Total	18		

Grado de precisión

Amplitud del intervalo de confianza del 80% alrededor de la estimación de la tendencia central.	6,51%<30
Grado de precisión	III

Inmueble tasable/datos

Ac	90,00
Pe	1,00
Ed	2,00
Ca	3,00
Pi	28,00
DPV	1.537,34

Estimación puntual/intervalo

Regressão Linear	
Valor Unitário	
Mínimo	2.021,86
Médio	2.088,73
Máximo	2.157,82
Valor Total	
Mínimo (3,20%)	181.967,19
Médio	187.985,98
Máximo (3,31%)	194.203,85
Intervalo Predição	
Mínimo (9,77%)	169.615,06
Máximo (10,83%)	208.346,65
Mínimo (IP)	1.884,61
Máximo (IP)	2.314,96
Campo de Arbítrio	
RL Mínimo	159.788,09
RL Máximo	216.183,88
Parâmetros	
Nível de Confiança	80%
Estimativa pela	Mediana

Intervalo de valores

Estimación central	187.985,98
Intervalo de confianza	
181.967,19	194.203,85
Intervalo de predicción	
169.615,06	208.346,65
Campo de arbitrio	
159.788,08	216.183,88
Valor arbitrado	187.985,98
Intervalo arbitrado	
181.967,19	194.203,85
Intervalo admisible	
181.967,19	194.203,85

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PH Arenas Park

Regressão Linear	
Valor Unitário	
Mínimo	2.169,09
Médio	2.216,06
Máximo	2.264,04
Valor Total	
Minimo (2,12%)	145.329,06
Médio	148.475,89
Máximo (2,17%)	151.690,86
Intervalo Predição	
Minimo (9,52%)	134.343,06
Máximo (10,52%)	164.095,49
Mínimo (IP)	2.005,12
Máximo (IP)	2.449,19
Campo de Arbítrio	
RL Mínimo	126.204,51
RL Máximo	170.747,27

USD 165.000,00

PH Vista Marina Tower

Regressão Linear	
Valor Unitário	
Mínimo	1.657,71
Médio	1.718,44
Máximo	1.781,41
Valor Total	
Minimo (3,53%)	174.059,30
Médio	180.436,61
Máximo (3,66%)	187.047,59
Intervalo Predição	
Minimo (9,89%)	162.594,43
Máximo (10,97%)	200.236,69
Mínimo (IP)	1.548,52
Máximo (IP)	1.907,02
Campo de Arbítrio	
RL Mínimo	153.371,12
RL Máximo	207.502,11

USD 170.000,00

PH Coral Tower

Regressão Linear	
Valor Unitário	
Mínimo	2.219,39
Médio	2.291,64
Máximo	2.366,24
Valor Total	
Minimo (3,15%)	179.770,79
Médio	185.622,69
Máximo (3,26%)	191.665,08
Intervalo Predição	
Minimo (9,77%)	167.484,99
Máximo (10,83%)	205.724,59
Mínimo (IP)	2.067,72
Máximo (IP)	2.539,81
Campo de Arbítrio	
RL Mínimo	157.779,28
RL Máximo	213.466,09

USD 200.000,00

PH Miradores Residences

Regressão Linear	
Valor Unitário	
Mínimo	2.053,43
Médio	2.110,72
Máximo	2.169,61
Valor Total	
Minimo (2,71%)	184.808,78
Médio	189.964,76
Máximo (2,79%)	195.264,58
Intervalo Predição	
Minimo (9,65%)	171.628,56
Máximo (10,68%)	210.259,93
Mínimo (IP)	1.906,98
Máximo (IP)	2.336,22
Campo de Arbítrio	
RL Mínimo	161.470,04
RL Máximo	218.459,47

USD 168.500,00

Is it possible to make the decision to invest just by analyzing the prices of the real estate?



What method can be used to help make the best possible real estate investment decision?

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Real estate investment and decision making with AHP

Table 6: Random sample of offers with total prices

Property	Construction area [m^2]	Parking Place [adim]	Age [year]	Quality [adim]	Floor [adim]	Distance to the valorizing pole [m]	Total price [UM]
PH Arenas Park	67	1	5	2	23	1.157,96	165.000,00
PH Vista Marina Tower	105	2	8	1	16	1.541,94	170.000,00
PH Coral Tower	81	2	5	3	26	782,71	200.000,00
PH Miradores Residences.	90	2	2	2	28	1.537,34	168.500,00

Own elaboration based on Camacaro Mock (2021)

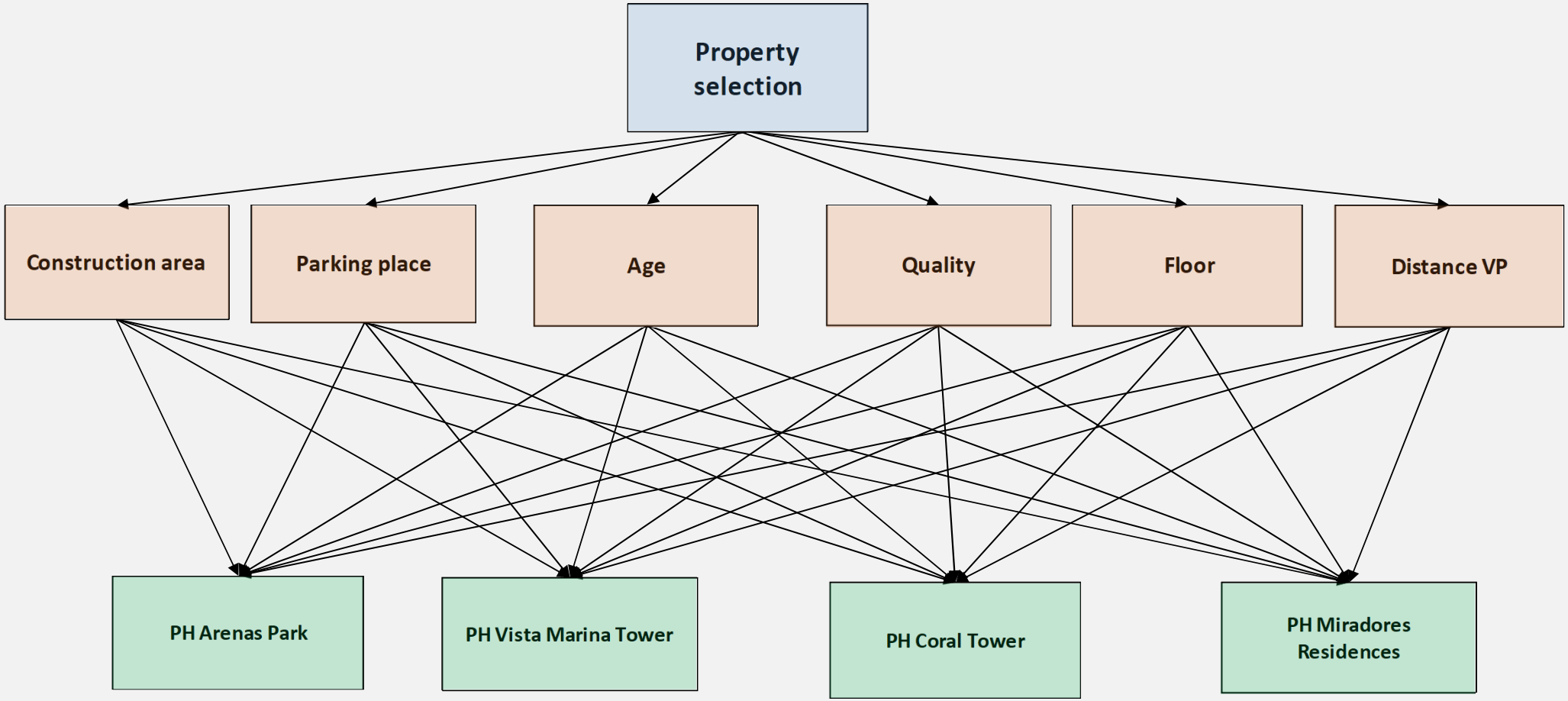

Property selection
Construction area
Parking place
Age
Quality
Floor
Distance VP
PH Arenas Park
PH Vista Marina Tower
PH Coral Tower
PH Miradores Residences

Table 7: Matrix of criteria comparisons

	Construction area	Parking place	Age	Quality	Floor	Distance VP
Construction area	1	5	3	3	4	3
Parking place	1/5	1	1/4	1/5	1/5	1/5
Age	1/3	4	1	2	3	3
Quality	1/3	5	1/2	1	3	3
Floor	1/4	5	1/3	1/3	1	2
Distance VP.	1/3	5	1/3	1/3	1/2	1
Sum	2,45	25,00	5,42	6,87	11,70	12,20

Own elaboration based on Camacaro Mock (2021))

Table 8: Matrix normalization, eigenvector, total vector and λ_{max}

	Normalization						Eigen vector	Total Vector	TV/EV
Ac	0,4082	0,2000	0,5538	0,4369	0,3419	0,2459	**0,3644**	2,46	6,74
Pe	0,0816	0,0400	0,0462	0,0291	0,0171	0,0164	**0,0384**	0,24	6,28
E	0,1361	0,1600	0,1846	0,2913	0,2564	0,2459	**0,2124**	1,46	6,89
C	0,1361	0,2000	0,0923	0,1456	0,2564	0,2459	**0,1794**	1,22	6,77
P	0,1020	0,2000	0,0615	0,0485	0,0855	0,1639	**0,1103**	0,71	6,48
DPV	0,1361	0,2000	0,0615	0,0485	0,0427	0,0820	**0,0951**	0,59	6,25
								λ_{max}	6,57

Own elaboration based on Camacaro Mock (2021)

The Consistency Index is calculated ***C.I.***:

$$C.I. = \frac{(\lambda_{max}) - n}{(n) - 1} = \frac{6{,}57 - 6}{6 - 1} = 0{,}11$$

We proceed to calculate the Consistency Ratio ***C.R.***, previously the Random Index (***R.I.***) is determined, entering in Table 3 for n=6; ***R.I.*** is equal to 1.24:

$$C.R = \frac{C.I.}{R.I.} = \frac{0{,}11}{1{,}24} = 0{,}09 < 0{,}10$$

Complies with Table 4

Table 9: Eigenvectors of the alternatives, criteria and general weighting vector

Alternative	*Ac*	*Pe*	*E*	*C*	*Pi*	*DP V*	Criteria vector	General vector	Ranking
PH Arenas Park	0,31	0,14	0,20	0,17	0,25	0,25	0,36	**0,2423**	3
PH Vista Marina Tower	0,20	0,29	0,12	0,05	0,17	0,19	0,04	**0,1554**	4
PH Coral Tower	0,26	0,29	0,20	0,61	0,28	0,37	0,21	0,3213	1
PH Miradores Residences	0,23	0,29	0,49	0,17	0,30	0,19	0,18	**0,2810**	2
							0,11		
							0,10		

Own elaboration based on Camacaro & Mock (2021)

Is it possible to substitute expert opinion in the pairwise comparison matrix of criteria and alternatives?



Can the statistical results of an MLRM replace the values of the Saaty Table?

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From MLRM and Criterion t*: the proposed replacement of expert opinion and Saaty's table in AHP.

Table 10: Criteria Comparison Matrix (t)*

Variable	*t**	*Ac*	*Pe*	*E*	*C*	*P*	*DPV*
Ac	11,9	1	5,38	4,33	6,69	2,80	3,56
Pe	2,21	0,16	1	0,80	1,24	0,52	0,66
E	2,75	0,23	1,247	1	1,55	0,65	0,82
C	1,78	0,15	0,81	0,65	1	0,42	0,53
P	4,25	0,36	1,92	1,56	2,39	1	1,27
DPV	3,34	0,28	1,51	1,21	1,88	0,79	1
	Sum	2,20	11,87	9,54	14,74	6,17	7,85

Own elaboration based on Camacaro & Mock (2021)

Table 11: Matrix normalization, eigenvector, total vector and λ_{max}

	Normalization						Eigen vector	Total vector	TV/EV
Ac	0,45	0,45	0,45	0,45	0,45	0,45	0,45	2,72	6,00
Pe	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,51	6,00
E	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,63	6,00
C	0,07	0,07	0,07	0,07	0,07	0,07	0,07	0,41	6,00
P	0,16	0,16	0,16	0,16	0,16	0,16	0,16	0,97	6,00
DPV	0,13	0,13	0,13	0,13	0,13	0,13	0,13	0,76	6,00
								λmax	6,00

Own elaboration based on Camacaro & Mock (2021)

The Consistency Index is calculated ***C.I.***:

$$C.I. = \frac{(\lambda_{max}) - n}{(n) - 1} = \frac{6 - 6}{6 - 1} = 0{,}00$$

We proceed to calculate the Consistency Ratio ***C.R.***, previously the Random Index (***R.I.***) is determined, entering in Table 3 for *n*=6; ***R.I.*** is equal to 1.24:

$$C.R = \frac{C.I.}{R.I.} = \frac{0{,}00}{1{,}24} = 0{,}00 < 0{,}10$$

Complies with Table 4[8]

Table 12: Eigenvectors of the alternatives, criteria and general weighting vector general

Alternative	*Ac*	*Pe*	*E*	*C*	*Pi*	*DPV*	Criteria vector	General vector	Ranking
PH Arenas Park	0,31	0,14	0,20	0,25	0,25	0,25	0,45	0,26	3
PH Vista Marina Tower	0,20	0,29	0,12	0,13	0,17	0,19	0,08	0,19	4
PH Coral Tower	0,26	0,29	0,20	0,38	0,28	0,37	0,10	0,28	1
PH Miradores Residences	0,23	0,29	0,49	0,25	0,30	0,19	0,07	0,27	2
							0,16		
							0,13		

Own elaboration based on Camacaro & Mock (2021)

Table 13: Eigenvectors of the alternatives, criteria and overall weighting vector

	Saaty's Table	Gauss Factor	Statistical (t^*)
PH Arenas Park	0,24	0,23	0,26
PH Vista Marina Tower	0,16	0,17	0,19
PH Coral Tower	0,32	0,29	0,28
PH Miradores Residences	0,28	0,32	0,27
Mean	0,25	0,25	0,25
Standard deviation	0,07	0,07	0,04
Coefficient of variation (C.V.)	28%	26%	17%

Own elaboration based on Camacaro (2022)

DISCUSSION

- **When MADMs are used with an empirical foundation, decision makers (DMs) are very likely to optimize the selection of the best alternative for their investment, business or venture. However, the solution provided by a multi-criteria method alone does not necessarily represent the final decision.**
- **Since the appearance of the popular multi-criteria AHP method, the fundamental scale of the Saaty Table has been widely investigated, with mixed opinions in the scientific community.**
- **The suggested methodology for the replacement of the pairwise comparison in the criteria and alternatives matrices by numerical matrices based on relationships between MLRM test statistics and the use of quantitative variables, requires the real estate consultant to have full knowledge of the behavior of real estate prices and the variables involved in their formation.**

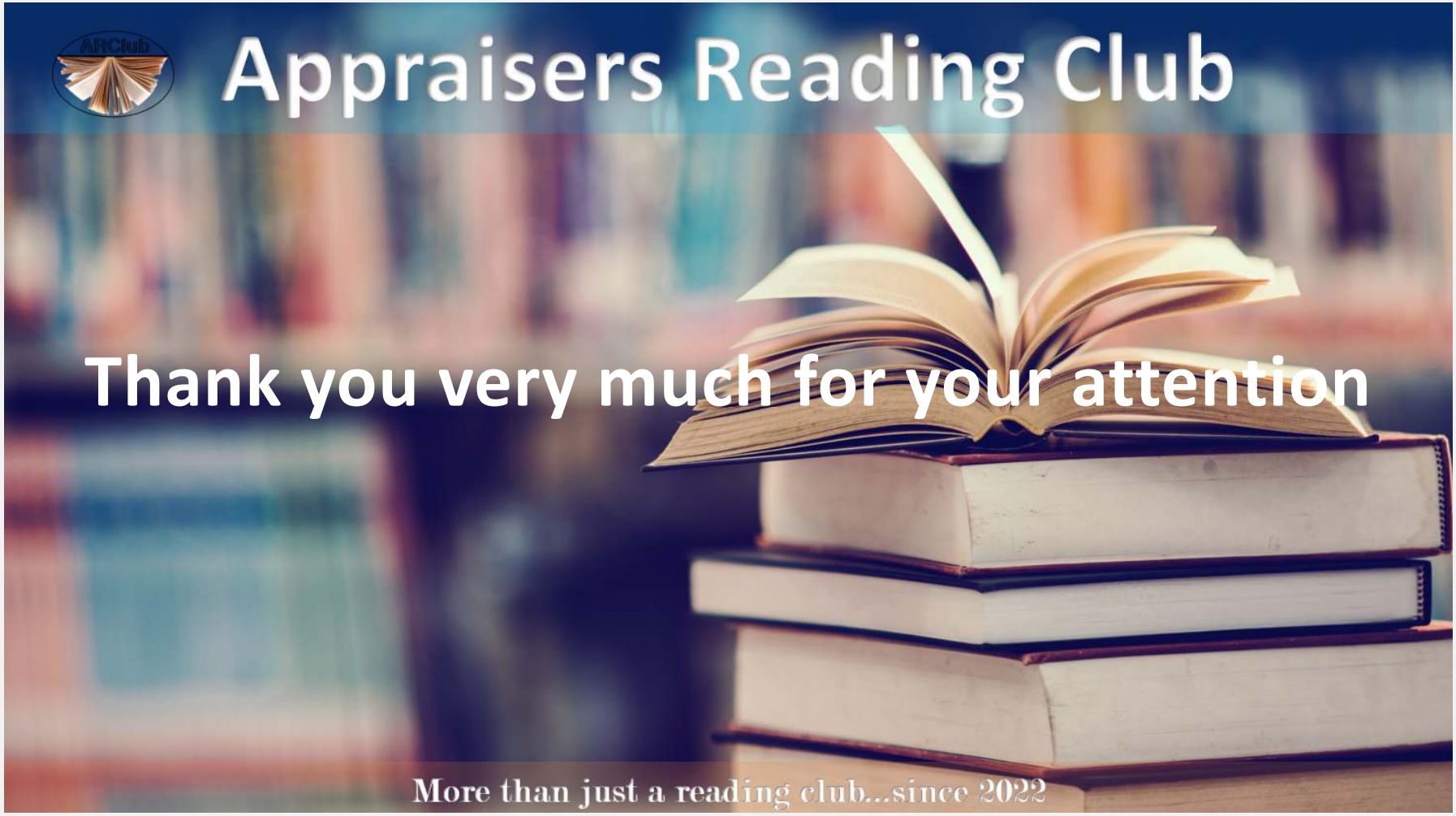
- **The figure of the real estate appraiser is key to analyze the databases, and with the use of scientific methodology generate the econometric models to extract the test statistics that empirically support the solutions when the AHP is used.**
- **Currently, there are several computer programs, such as Expert Choice and Super Decisions, which facilitate access to mathematical calculations that allow consolidating the knowledge and development of the AHP multi-criteria method. There are also several technological tools that allow, in this era of Big Data, the processing of data to provide solutions when it is necessary to make a decision using multiple criteria.**

CONCLUSION

- **The use of the matrix of the relationships between the *t** statistics associated with the MLRM variables replaces the expert judgments in the surveys based on the Saaty Table, allowing decisions to be made with a sufficient degree of objectivity, thus mitigating the subjectivity in the heuristic solution provided by the AHP.**
- **For the analysis of real estate offers, the *t** Criterion represents a valid alternative and serves to support the work of real estate professionals, with the advice of expert appraisers, when using the AHP to optimize the marketing of real estate products and satisfy the requirements of potential buyers for the best investment decision.**



… And finally, the happy couple was able to make the best decision for their real estate investment, which may be the only and the most important one they will make in their long life...


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