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# Why are AHP results not binding for appraising real estate?

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## ABSTRACT

Using the method of reduction to absurdity, an initial hypothesis was considered: the results of the general weighting vector of AHP are not linked to real estate market prices, as opposed to the alternative hypothesis that they are linked. To demonstrate this, a "real fable" was used to explain the valuation of a medieval castle. The first example was based on three criteria: quality of the environment, quality of the castle, and state of conservation; and six alternatives, five comparables plus the property being appraised (3C+6A), completing three more cases. Using AHP, the elements of the global vector of the alternatives were determined. With the offer prices and the use of RV, the value of the property and the values of the comparables were estimated for comparative analysis. The Non-Ratio Reciprocity Test (NRRT) was used, obtaining significant percentage differences between the values and prices of the comparables, not guaranteeing reciprocity to validate the initial estimates of the appraised property value. In the second example, three different criteria were used: strength, security, and access; the same six alternatives were used (3C+6A). It was possible to demonstrate the inconsistency of the results due to the disparity of the values obtained in all cases of the two examples for the appraised property, which does not allow for guaranteeing reliable market values. For the third example, the database simulation was performed based on the information from the original sample, with 8 explanatory variables, to obtain 69 data points. Econometric modeling was carried out, demonstrating that there is no correlation between the explained variable (unit price) and the explanatory variables as a whole, much less individually, therefore, the results are not reliable. Despite the non-fulfillment of the basic assumptions of MRLM, point and interval estimation was done. These values were similar to those obtained in the second example, using AHP+RV. With this evidence, the analyzed proposal does not guarantee market values; therefore, the initial hypothesis is correct.

**Key words**: AHP, models, valuation, real estate, MRLM.

## 1. INTRODUCTION

The science of valuation integrates elements of economics, finance, statistics and law to determine the value of property and assets. Throughout history, this discipline has evolved significantly, adapting to economic changes and the needs of society.

Numerous procedures and methods have been proposed from different areas of knowledge, and their adoption and adaptation have enriched research in this field. This has made valuation an essential discipline for study and practice in the modern global economy.

An example of this adoption is the interdisciplinary field called Operations Research, which has a branch that explicitly evaluates multiple criteria, which is the Multiple Criteria Decision Making (MCDM), being one of the established methods, the Analytic Hierarchy Process (AHP) [1], that Thomas Saaty developed in the 1970s and 1980s.

Since its appearance, the method proposed by Saaty and his collaborators has been highly praised and popular among its users. It is a method created to solve problems of selection of alternatives for complex decision making.

At the beginning of the 21st century, professors at the Universidad Politécnica de Valencia [2] (Spain) introduced a procedure using the AHP plus a Valuation Ratio as an alternative valuation method. Today, Latin American appraisers have adopted it as their own, without any of the advocates of the use of the AHP method[3], that have won awards at valuation congresses of the UPAV[4], have presented empirical evidence to support the use of the valuation ratio to "leverage" the AHP ranking.

Although I knew about the proposal since its beginnings, I was never interested in studying it because it seemed to me a rather subjective method to be applied to real estate valuation, and because I was caught by the teaching of the Brazilian school. However, years later, seeing the boom in the use of this procedure among Latin American appraisers, I was curious to know why they considered it as a reliable alternative procedure to value real estate.

I took the initiative to study the origins of MCDA, in particular the AHP multi-criteria method. As a result of my research, I made a first observation on the results of the AHP + Valuation Ratio, introducing the Non-Ratio Reciprocity Test (NRRT), which showed that the elements of the global vector of alternatives are not binding to determine deterministic market values.

Also, with Monte Carlo scenario simulations it was shown that the probabilistic estimation of the results does not generate reciprocity between the values of the analyzed properties when they are removed or increased comparable in the sample analyzed through AHP and the valuation ratio. The application of the procedure is limited to the use of small samples, most of the times they do not exceed six comparables + six variables, not complying with the contents of the valuation standards[5], therefore, the results are punctual but not valid, since they do not allow generalization to obtain market values.

The purpose of this work is to formalize this research, therefore, in the Methodology section, the theoretical bases of the Direct Comparative Method of Market Data, based on the Brazilian valuation standards ABNT NBR 14653-2, the Analytic Hierarchy Process (AHP) and the Method of Reduction to the Absurd are succinctly presented. In the results, the hypotheses considered are presented, followed by a demonstration using a "real fable" where three examples are developed,

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[1] See the video 5 TMDII - Information about *Analytic Hierarchy Process* (AHP) in: https://youtu.be/oRVA-2fhrM4

[2] Multicriteria Group UPV https://www.upv.es/contenidos/VMULTIC/infoweb/vmultic/info/1068975normalc.html

[3] A review was done at ISAHP from 1998 to 2022 and there is no evidence of such work. https://www.isahp.org/proceedings/

[4] The Pan-American Union of Valuation Associations (UPAV) is an organization that brings together various valuation associations from different Latin American and European countries.

[5] Orden ECO/805/2003, of March 27, 2003, on rules for the valuation of real estate and certain rights for certain financial purposes (Spain).

using the AHP + Valuation Ratio and the Multiple Linear Regression Model (MLRM). Then, the Discussion and Conclusion sections are presented, where the formality of the methodological proposal resulting from the research work is presented.

## 2. METHODOLOGY

For the development of this article, it is very important to briefly explain the Analytic Hierarchy Process (AHP), the Comparative Market Data Method and the Reduction to Absurdity Method.

### 2.1 Analytic Hierarchy Process -AHP

Thomas Saaty introduced the Analytic Hierarchy Process (AHP) in the 1970s and the Analytic Network Process (ANP) more recently. Among his co-authors and colleagues are Ernest Forman and Luis Vargas. Saaty is one of the most successful people in the field of information management and communications[6] in the dissemination of his method for complex decision making.
According to *Moreno Jiménez et al* (2019), the Analytic Hierarchy Process (AHP) is a multicriteria decision technique that combines tangible and intangible aspects to obtain, on a ratio scale, the priorities associated with the alternatives of the problem. In an environment of certainty, the AHP provides the possibility of including quantitative data related to the decision alternatives.
The advantage of the AHP is that it additionally allows the incorporation of qualitative aspects that are usually left out of the analysis due to their complexity to be measured, but that may be relevant in some cases. The main characteristics of this approach are the modeling of the problem by means of a hierarchical structure, the use of paired comparisons to incorporate the preferences of the decision maker, and the obtaining of a ratio scale that is valid for complex decision making.
According to Saaty, making a decision in an organized way to generate priorities requires breaking down the decision into the following steps:

1. Define the problem and determine the type of knowledge being sought.
2. Structure the decision hierarchy from the top with the decision objective, then the objectives from a broad perspective, through the intermediate levels (criteria on which subsequent elements depend) to the lowest level (which is usually a set of alternatives).
3. Construct a set of pairwise comparison matrices. Each element of a higher level is used to compare the elements of the level immediately below with respect to it.
4. Use the priorities obtained from the comparisons to weight the priorities at the next lower level. This is done for each element. Then, for each item at the lower level, add their weighted values and obtain their overall priority. Continue this process of weighting and summing until you get the final priorities of the alternatives at the lowest level.

AHP is an effective tool for making complex decisions, as it decomposes the problem into manageable parts and quantifies the decision-makers' preferences. It provides a systematic methodology for evaluating and comparing multiple criteria and alternatives. Although AHP is useful, its results are not binding and must be supplemented with other methods and qualitative judgments, especially in real estate valuation where contextual factors may be significant. Consistency of comparisons is crucial; inconsistent decisions can lead to unreliable results.
In summary, AHP facilitates well-informed, consistent and transparent decisions, although it should always be considered as a part of the broader decision-making process.

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[6]According to the content of the Part of the history of Multiple Criteria Decision Making (MCDM) taken from The International Society on Multiple Criteria Decision Making website. Retrieved from: https://www.mcdmsociety.org/content/short-mcdm-history-0

## 2.2 Direct Comparative Market Data Method

In point 8.2.1.4.3 of the ABNT [7] NBR 14653-2, It refers to the scientific treatment and expresses that whatever models are used to infer market behavior and the formation of its values, their assumptions must be duly explained and tested.
When talking about factors that affect the market value of real estate, it is necessary to use a model, and this is nothing more than a simplified representation of market reality. Econometric models are a particular class of models that aim to quantify relationships between variables based on economic laws that support them.
In a broad sense, it can be said that econometric analysis deals with developing models through which hypotheses relating to economic systems can be verified.
The models that relate the price of a property with its characteristics are known as hedonic price models that can be expressed through the following function:

$$P = f(L, E, T) + \varepsilon \quad (1)$$

Where:

$P$ : price of the good based on its attributes.
$f$ : functional form.
$L$ : localization variables.
$E$ : structural variables.
$T$ : temporary variables.
$\varepsilon$: errors.

To explain the behavior of real estate prices based on a population of $m$ observed prices ($Y_i$) considering the influential characteristics ($X_{ij}, j=1,...,k$) the multiple linear regression model is used:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + ... + \beta_k X_{ik} + \varepsilon_i;\ i=1,...,m \quad (2)$$

Where:

$Y_1,...,Y_m$: dependent variable (price of real estate).
$X_{1k},\ldots,X_{mk}$: independent variables (attributes of the real estate).
$\beta_0,\ldots,\beta_k$: model parameters (shadow prices).
$\varepsilon_1,\ldots,\varepsilon_m$: random errors of the model.

In the linear regression model for the representation of the real estate market, the dependent variable is expressed as a linear combination of the independent variables, in original or transformed scale, and the respective estimates of the population parameters, including the errors.

Based on the sample extracted from the market, the estimation of the regression model parameters is done using the least squares method or the maximum likelihood method. The modeling must expose the hypotheses regarding the behavior of the dependent and independent variables, based on the valuation engineer's knowledge of the market, when the null or alternative hypotheses are formulated for each population parameter.

When regression models are used, the content of the standard highlights the need to observe the basic assumptions, mainly in relation to normality, homoscedasticity, non-multicollinearity, non-

[7] ABNT stands for the Brazilian Association of Technical Standards, the body responsible for standardization in Brazil. The acronym NBR refers to Brazilian Standards, in this case, NBR-14653 Part 2. Appraisals of Urban Assets.

autocorrelation, independence or non-existence of atypical points and specification errors, with the objective of obtaining unbiased, efficient and consistent values.

### 2.3 Method of reduction to absurdity[8]

Reduction to absurdity (or *reductio ad absurdum*) is a technique of argumentation and demonstration used to establish the truth of a proposition or situation by showing that assuming the opposite leads to a contradiction or an absurd result. It can be applied in various fields, mainly in logic, mathematics, philosophy and science, but also in everyday situations and legal argumentation.
It consists of temporarily assuming the opposite of what one wants to prove, and then demonstrating that this assumption leads to a contradiction or an absurd result. This implies that the initial assumption is false, therefore, the original statement must be true.

## 3. RESULTS

First, we start from the hypotheses of the reduction to absurdity method:

- Initial hypothesis: the results of the general weighting vector of the AHP are not linked to real estate market prices.
- The alternative hypothesis is: The results of the general weighting vector of the AHP are linked to real estate market prices.

The method was developed and tested using the following fable: One day, a king[9] from a European country decided to give the crown prince a new castle, with all the luxuries and comforts. Using web scraping[10], applied to royal real estate portals, he obtained a small database: a sample of five properties based on three characteristics: quality of the environment, quality of the castle and state of conservation. The other characteristics were quite similar to the reference properties. He was also firm in revising the prices offered.
Now, which castle to buy? It was a decision that had to be made quickly because the market for medieval castles was very desirable and the selected properties could be sold quickly. A nobleman found out about the royal intentions, although his castle (Castle N, as it will be called later) built on a 15.00 ha plot and with a construction area of 1,500.00 $m^2$, which was not for sale at that time, decided to offer it to His Majesty, but there was a problem: he had not thought about the price to offer.
So she took the plunge, went to the Royal Castle and talked to the King about her proposal. The latter was interested in the property, because in his youth he had gone to that castle a lot, in addition to having fun with his childhood friends, so the castle seemed very familiar to him, and there he also met his first love... unrequited, unexpected details of royalty. Interested as he was, the King asked him the normal, formal question, as in any negotiation: What is the price of your castle? The nobleman quickly replied that he did not know. Adding that the emotional value is very high, as high as my debts, he argued, so he told His Majesty to make an estimate of the value, which he would analyze to decide on the sale price, the negotiation would surely be done, noble word of honor.

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[8] In this article you can read more about this topic: https://definicion.com/reductio-ad-absurdum/
[9] Essay inspired by: *Malba Tahan* (1938). "The Man Who Calculated". Chapter 16 "Legend about the game of chess by *Beremís Samir*".
[10] Web scraping is a technique used to extract data from websites in an automated manner. It involves the use of software or scripts that crawl web pages, retrieve the desired information, and store it in a structured format, such as a spreadsheet or database.

Very well, said the King, bowing to the nobleman. In my court there is an expert appraiser, he continued, who will be able to correctly estimate the most probable price of your castle so that the best decision for the parties can be made. Indeed, he called the Royal Appraiser who immediately accepted the assignment to carry out the Appraisal work. Via email, the King sends the market sample to the appraiser so that he can perform the calculations and determine the market value of the property, complying with the royal appraisal standards. This table contains the information provided by His Majesty[11]:

Table 1: Comparables of the medieval castle market in Europe [12]

| Source of information | Comparable | Total Price [UM] | Land area [ha] | Construction area [m²] | Unit price [UM/m²] |
|---|---|---|---|---|---|
| www.real2.com | Castle 1 | 2,475,000.00 | 20.00 | 1,500.00 | 1,650.00 |
| www.majestic.com | Castle 2 | 3,000,000.00 | 15.00 | 2,500.00 | 1,200.00 |
| www.castle.com | Castle 3 | 5,512,500.00 | 30.00 | 3,500.00 | 1,575.00 |
| www.luxury.com | Castle 4 | 4,080,000.00 | 25.00 | 2,000.00 | 2,040.00 |
| www.nobile.com | Castle 5 | 12,400,000,00 | 20.00 | 4,000.00 | 3,100.00 |

It is worth remembering the characteristics of the properties that the King had considered to select the sample, so the appraiser had to consider them in his appraisal work, otherwise it would cause a "displeasure" to the King, since his opinion as a businessman was highly respected throughout the world, and even more so when he began the study for the selection of the property as a family investment.

The appraiser, a market man, had no doubts about which method to apply, the Comparative Market Data Method, but because of the size of the sample, although he knew that there was much more information available, it was not possible to apply scientific treatments to explain real estate prices. The deadline was two days, and he wondered what to do? It was then that he remembered a proposal brought from Europe to America, based on the use of a valuation ratio using the elements of the global vector of alternatives of the Analytic Hierarchy Process (AHP).

In order to assume this valuation procedure, he considered, according to his criteria, three convincing elements: size of the sample, characteristics of the properties and time of delivery of the report. Once the decision on the procedure to be used had been made, he quickly called upon five expert appraisers from the realm to help him with their opinions to define the reciprocal matrices of pairwise comparisons of the criteria and of the alternatives by criteria.

After an intense day of analysis that ended in the early hours of the morning, they did not reach a unanimous decision on the comparisons of the characteristics, so they used the geometric mean to calculate the elements and thus guarantee reciprocity in the matrices. In the case of the criteria matrix, this was the solution provided by the group of experts (standardized synthesis[13]), based on Saaty's fundamental scale:

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**11** The unit prices in the following table are obtained by dividing the total price of the comparable by the construction area.

[12] The sources consulted are fictitious. The values of the variables indicated in the table are also fictitious, except for the unit values, which were taken from the example of the urban property in the book "New valuation methods Multi-criteria models" by Jerónimo Aznar and Francisco Guijarro (2012) p. 92 to 95. The values of the reciprocal matrices of paired comparisons of the criteria and the alternatives by the criteria of the first example of this essay are also from this reference.

[13] *Solomon (2016*). Normalizing priorities creates a dependency between them. However, by removing an old alternative or inserting a new one, normalized priorities can cause illegitimate changes in the rank of the alternatives, which is known as Rank Reversal-RR. The Rank Reversal-RR video on AHP is available on the Mundo valor channel. You can access it through this link https://youtu.be/5TkAxT4kDPA

Table 2: Reciprocal matrix of criteria and priority vector [14]

| Criteria | Quality of the environment | Quality of the castle | State of conservation | Vector priority |
|---|---|---|---|---|
| Quality of the environment | 1 | 1/3 | 1/4 | 0.12196 |
| Quality of the castle | 3 | 1 | 1/2 | 0.31962 |
| State of conservation | 4 | 2 | 1 | 0.55842 |
| Consistency ratio = | 1.76 % | | | |

As can be seen, the appraisers considered the state of conservation to be the criterion or characteristic with the highest priority, followed by the quality of the castle, and then the quality of the environment. To determine the priority vector of the alternatives by criteria, the appraisers provided their judgments contained in the following tables. Starting with the characteristic, quality of the environment:

Table 3: Reciprocal matrix of the alternatives for the quality of the environment and normalized priority vector

| Quality of the environment | Castle 1 | Castle 2 | Castle 3 | Castle 4 | Castle 5 | Castle N | Normal priority vector |
|---|---|---|---|---|---|---|---|
| Castle 1 | 1 | 3 | 3 | 1/3 | 1/5 | 1/3 | 0.0825 |
| Castle 2 | 1/3 | 1 | 1 | 1/7 | 1/9 | 1/7 | 0.0328 |
| Castle 3 | 1/3 | 1 | 1 | 1/7 | 1/9 | 1/7 | 0.0328 |
| Castle 4 | 3 | 7 | 7 | 1 | 1/3 | 1 | 0.2069 |
| Castle 5 | 5 | 9 | 9 | 3 | 1 | 3 | 0.4381 |
| Castle N | 3 | 7 | 7 | 1 | 1/3 | 1 | 0.2069 |
| Consistency ratio = | 1.96 % | | | | | | |

It can be observed that Castle 5, due to its highest value among the other elements of the normalized priority vector for the quality of the environment criterion, is the one preferred by the experts, that is, they consider that its environment is much more important for the decision when compared to the environments of the other castles. To determine the priority vector of the alternatives for the Quality of the castle, the appraisers reached this conclusion, which is expressed in the following table:

Table 4: Reciprocal matrix of the alternatives for the quality of the castle and normalized priority vector

| Quality of the castle | Castle 1 | Castle 2 | Castle 3 | Castle 4 | Castle 5 | Castle N | Normal priority vector |
|---|---|---|---|---|---|---|---|
| Castle 1 | 1 | 3 | 1/3 | 1 | 1/5 | 1 | 0.0890 |
| Castle 2 | 1/3 | 1 | 1/7 | 1/3 | 1/9 | 1/3 | 0.0343 |
| Castle 3 | 3 | 7 | 1 | 3 | 1/3 | 3 | 0.2355 |
| Castle 4 | 1 | 3 | 1/3 | 1 | 1/5 | 1 | 0.0890 |
| Castle 5 | 5 | 9 | 3 | 5 | 1 | 5 | 0.4630 |
| Castle N | 1 | 3 | 1/3 | 1 | 1/5 | 1 | 0.0890 |
| Consistency ratio = | 1.24 % | | | | | | |

It can be observed that Castle 5, due to its highest value among the other elements of the normalized priority vector for the quality of the castle criterion, is the one preferred by the experts,

[14] In all the tables of the reciprocal matrices, at the bottom, appears the consistency ratio that is used to measure the consistency of the judgments of the paired comparisons. On the Mundo Valor channel there is the video “Calculation of the Consistency Ratio” that you can access through this link https://youtu.be/bCJr_THM-2Y

that is, they consider that its quality is much more important for the decision when compared to the quality of the other castles.
In order to determine the priority vector of the alternatives by state of conservation, the appraisers reached this conclusion, which is expressed in the following table:

Table 5: Reciprocal matrix of the alternatives by state of conservation and normalized priority vector

| State of conservation | Castle 1 | Castle 2 | Castle 3 | Castle 4 | Castle 5 | Castle N | Normal priority vector |
|---|---|---|---|---|---|---|---|
| Castle 1 | 1 | 5 | 3 | 1 | 1 | 3 | 0.2559 |
| Castle 2 | 1/5 | 1 | 1/3 | 1/5 | 1/5 | 1/3 | 0.0428 |
| Castle 3 | 1/3 | 3 | 1 | 1/3 | 1/3 | 1 | 0.0947 |
| Castle 4 | 1 | 5 | 3 | 1 | 1 | 3 | 0.2559 |
| Castle 5 | 1 | 5 | 3 | 1 | 1 | 3 | 0.2559 |
| Castle N | 1/3 | 3 | 1 | 1/3 | 1/3 | 1 | 0.0947 |

Consistency ratio = 0.93 %

Again, according to the experts' judgment, Castle 5 was the most important along with Castles 1 and 4, as they had the same value among the elements of the normalized priority vector.
Once the priority vectors of the criteria and the normalized priority vectors of the alternatives by the criteria have been obtained, the table is presented where the results of the calculations for the selection of the best alternative are, that is, the global vector with the positions (ranking) based on the judgment of the expert evaluators:

Table 6: Global vector results and ranking of alternatives

| Castle | Quality of the environment | Quality of the castle | State of conservation | Vector priority criteria | Normalized global vector | *Ranking* |
|---|---|---|---|---|---|---|
| Castle 1 | 0.08247 | 0.0890 | 0.2559 | 0.12196 | 0.1814 | 3 |
| Castle 2 | 0.03282 | 0.0343 | 0.0428 | 0.31962 | 0.0389 | 6 |
| Castle 3 | 0.03282 | 0.2355 | 0.0947 | 0.55842 | 0.1322 | 4 |
| Castle 4 | 0.20688 | 0.0890 | 0.2559 | | 0.1966 | 2 |
| Castle 5 | 0.43813 | 0.4630 | 0.2559 | | 0.3443 | 1 |
| Castle N | 0.20688 | 0.0890 | 0.0947 | | 0.1066 | 5 |

As can be seen, Castle 5 was the one that obtained the highest value among the elements of the normalized global vector (0.3443), being number 1 in the ranking. As for Castle N, with a value of 0.1066, it occupied the fifth position in the selection process.
If a decision had to be made, the Castle 5 option should be considered, but this would not necessarily be definitive because the Benefits, Costs, Opportunities and Risks would have to be analyzed for the final decision.
Up to this point, the AHP has been correctly applied in its original form, and for which it was idealized in the 1970s by its author, and today, almost 50 years later, it continues to be studied by the scientific community as an alternative to empirically support complex decision-making.
The appraiser, having seen the results and the quick way he had done it, felt the need to talk to the King and so he was received at the Royal Castle. Reunited, he explained to the King that, if he had to make a decision today, the best decision would be Castle 5, of course following his recommendations on the characteristics that should underpin the solution to the problem.

The King reviewed the tables and seemed to agree, but Castle 5 was the most expensive and he commented: “And as you well know, not everything that glitters is gold.” But how much is the price of Castle N? That is the one that really interests me. How much can I offer the noble lord for the castle?

The appraiser tells him that he will do the calculations shortly and will return again with the information on the value of the property. Indeed, he arrived at his office and proceeded to use a mathematical artifice called "Valuation Ratio" which is based on the existence of an "implicit" relationship, but not empirically demonstrated, between the elements of the global vector of the alternatives (comparable castles) and the prices offered for the properties. The results are shown in the following table:

Table 7: Castle N Unit Value Results based on Valuation Ratio

| Castle | Unit Value [UM/$m^2$] | Vvg [adim] | Valuation Ratio [UM/$m^2$] | Vvg N [adim] | Unit Value N [UM/$m^2$] |
|---|---|---|---|---|---|
| Castle 1 | 1,650.00 | 0.1814 | 10,706.17 | 0.1066 | 1,141.17 |
| Castle 2 | 1,200.00 | 0.0389 | | | |
| Castle 3 | 1,575.00 | 0.1322 | | | |
| Castle 4 | 2,040.00 | 0.1966 | | | |
| Castle 5 | 3,100.00 | 0.3443 | | | |
| Total | 9,565.00 | 0.8934 | | 1.0000 | |

From this table, the unit value of the Castle N of 1,141.77 UM/$m^2$ was extracted, therefore, the total value of the property was calculated:

Table 8: Total value of the Castle N[15]

| Castle | Construction area [$m^2$] | Unit Value [UM/$m^2$] | Total Value [UM] |
|---|---|---|---|
| Castle N | 1,500.00 | 1,150.00 | 1,725,000.00 |

The appraiser exclaimed, "How wonderful! How quickly I was able to perform the calculations." Now he prepared the royal report to be presented to His Highness so that he could make the best decision. The King received him in his royal office, and indeed, he gave him the corresponding report.

His Majesty read it carefully and stated that the price seemed reasonable to him, but again he was struck by the fact that Castle 5, the one with the best ranking, was the one with the highest unit price among the offers.

So he thanks the respected appraiser for his advice, and says that with more time he will resume reading the appraisal report and let him know any observations, if any.

While the appraiser was doing other work, the King continued to check the market daily in case another interesting offer appeared. He had noticed that Castle 2 was no longer for sale, so he wrote a message to the appraiser asking him to perform the calculations again without considering the aforementioned property, to analyze what effects it could have on the selection of the alternative, and how it could affect the price of the appraisable property.

After performing the calculations, the appraiser obtained the following results:

[15] The unit value of 1,141.17 UM/$m^2$ was rounded to 1,150.00 UM/$m^2$ to obtain whole numbers and improve the presentation of the example.

Table 9: Normalized priority vector of the alternatives by criteria, priority vector of the criteria, global vector and ranking.

| Castle | Quality of the environment | Quality of the castle | State of conservation | Vector priority criteria | Normalized global vector | Ranking |
|---|---|---|---|---|---|---|
| Castle 1 | 0.0812 | 0.0876 | 0.2727 | 0.12196 | 0.190210 | 3 |
| Castle 3 | 0.0338 | 0.2364 | 0.0909 | 0.31962 | 0.130446 | 4 |
| Castle 4 | 0.2056 | 0.0876 | 0.2727 | 0.55842 | 0.205383 | 2 |
| Castle 5 | 0.4738 | 0.5007 | 0.2727 | | 0.370109 | 1 |
| Castle N | 0.2056 | 0.0876 | 0.0909 | | 0.103852 | 5 |
| Consistency ratio = | 2.28 % | 0.94 % | 0.00 % | | | |

With this information, the appraiser proceeded to calculate the value of Castle N, for which he presented the following table:

Table 10: Castle N Unit Value Results based on Valuation Ratio (without Castle 2)

| Castle | Unit Value [UM/m$^2$] | Vvg [adim] | Valuation Ratio [UM/m$^2$] | Vvg N [adim] | Unit Value N [UM/m$^2$] |
|---|---|---|---|---|---|
| Castle 1 | 1,650.00 | 0.1902 | 9,334.40 | 0.1039 | 969.40 |
| Castle 3 | 1,575.00 | 0.1304 | | | |
| Castle 4 | 2,040.00 | 0.2054 | | | |
| Castle 5 | 3,100.00 | 0.3701 | | | |
| Total | 8,365.00 | 0.8961 | | 1.0000 | |

From this table the unit value of Castle N of 969.40 UM/m$^2$ is extracted, therefore, the total value of the property is calculated:

Table 11: Total value of Castle N (excluding Castle 2)

| Castle | Construction area [m$^2$] | Unit Value* [UM/m$^2$] | Total Value [UM] |
|---|---|---|---|
| Castle N | 1,500.00 | 970.00 | 1,455,000.00 |

* Rounded

The appraiser observed that the difference in value of the appraisable property between the solution when Castle 2 was included and without considering Castle 2, was -270,000.00 UM, that is, -15.65%. So, he reflected and asked himself: being the King so expert in business, will this result "make him happy" as an investor? In any case, he put together the report and took it back to the King. Upon seeing the results, the King exclaimed: "If we interpret the results, Castle 2, which was the one that occupied No. 6 in the initial ranking and with the lowest unit price, influenced Castle N to have a lower unit value. Why this change?

The king exclaimed: «Could you be so kind, my dear and noble appraiser, to review the calculations? But not only those you have already made, but also to analyze other cases such as: leaving alternative 2 and not considering alternative 4, and since I got another offer, I would like you to include it with all the others, obtain the results and analyze them together».

The King asks him: Did I explain myself sufficiently? Do you understand my requirements? The answers were two convinced "yes, your Highness."

The appraiser, very interested in resolving this controversial situation due to the different real estate values already mentioned, proceeded to make the calculations obtaining the following results, for the case of the inclusion of Castle 2 and the exclusion of Castle 4:

Table 12: Normalized priority vector of the alternatives by criteria, priority vector of the criteria, global vector and ranking.

| Castle | Quality of the environment | Quality of the castle | State of conservation | Vector priority criteria | Normalized global vector | *Ranking* |
|---|---|---|---|---|---|---|
| Castle 1 | 0.1117 | 0.0983 | 0.3434 | 0.12196 | 0.236831 | 2 |
| Castle 2 | 0.0435 | 0.0388 | 0.0551 | 0.31962 | 0.048475 | 5 |
| Castle 3 | 0.0435 | 0.2527 | 0.1290 | 0.55842 | 0.158130 | 3 |
| Castle 5 | 0.5277 | 0.5119 | 0.3435 | | 0.419759 | 1 |
| Castillo N | 0.2735 | 0.0983 | 0.1290 | | 0.136805 | 4 |
| Consistency ratio = | 2.20 % | 0.94 % | 2.20 % | | | |

With this information, the appraiser proceeded to calculate the value of Castle N, for which he presented the following tab:

Table 13: Results of the Unit Value of Castle N based on the Valuation Ratio (without Castle 4)

| Castle | Unit Value [UM/m$^2$] | Vvg [adim] | Valuation Ratio [UM/m$^2$] | Vvg N [adim] | Unit Value N [UM/m$^2$] |
|---|---|---|---|---|---|
| Castle 1 | 1,650.00 | 0.2368 | 8,717.61 | 0.1368 | 1,192.61 |
| Castle 2 | 1,200.00 | 0.0485 | | | |
| Castle 3 | 1,575.00 | 0.1581 | | | |
| Castle 5 | 3,100.00 | 0.4198 | | | |
| Total | 7,525.00 | 0.8632 | | 1.0000 | |

From this table, the unit value of the Castle N of 1,192.61 UM/m$^2$ is extracted, therefore, the total value of the property is calculated:

Table 14: Castle N Unit Value Results based on Valuation Ratio (without Castle 4)

| Castle | Construction area [m$^2$] | Unit Value* [UM/m$^2$] | Total Value [UM] |
|---|---|---|---|
| Castillo N | 1,500.00 | 1,200.00 | 1,800,000.00 |

*Rounded

The appraiser observes that the unit value is similar to the one obtained with all the comparables, but he does not dare to make conclusions until he makes the other case, considering one more alternative, the Castillo 6 with a land area of 25 ha, construction area of 1,000.00 m2 and a total price of 3,000,000.00 UM. Again, he consulted with his valuator friends, they generated the paired comparisons considering the new alternative for the reciprocal matrix of the alternatives by criteria, obtaining the following results:

Table 15: Normalized priority vector of the alternatives by criteria, priority vector of the criteria, global vector and ranking.

| Castle | Quality of the environment | Quality of the castle | State of conservation | Vector priority criteria | Normalized global vector | Ranking |
|---|---|---|---|---|---|---|
| Castle 1 | 0.1190 | 0.1002 | 0.2168 | 0.12196 | 0.1676 | 2 |
| Castle 2 | 0.0266 | 0.0311 | 0.0342 | 0.31962 | 0.0323 | 7 |
| Castle 3 | 0.0266 | 0.1953 | 0.0853 | 0.55842 | 0.1133 | 5 |
| Castle 4 | 0.1548 | 0.0727 | 0.1817 | | 0.1436 | 4 |
| Castle 5 | 0.3435 | 0.3955 | 0.2378 | | 0.3011 | 1 |
| Castle 6 | 0.1612 | 0.1215 | 0.1588 | | 0.1472 | 3 |
| Castle N | 0.1684 | 0.0837 | 0.0853 | | 0.0949 | 6 |
| Consistency ratio = | 10.96 % | 4.40 % | 6.79 % | | | |

With these results, the appraiser completed the information and took charge of analyzing the ranking. He made the following comparative table:

Table 16: Comparison of ranking of alternatives by case

| Castle | Original case 6A | Case 5A (-2) | Case 5A (-4) | Case 7A (+6) |
|---|---|---|---|---|
| Castle 1 | 3 | 3 | 2 | 2 |
| Castle 2 | 6 | NA | 5 | 7 |
| Castle 3 | 4 | 4 | 3 | 5 |
| Castle 4 | 2 | 2 | NA | 4 |
| Castle 5 | 1 | 1 | 1 | 1 |
| Castle 6 | NA | NA | NA | 3 |
| Castle N | 5 | 5 | 4 | 6 |

Although in the first cases the general position of the alternatives is maintained, with Castillo 5 being the highest priority in all cases, in the case of the 7 alternatives there is an inversion of rank among the other alternatives, for example, Castillo 1 was the third option for the first two cases, then moved to second place for the following two cases, it is evident that there is an rank reversal[16]. As for Castle N, it occupied the following positions 5/6, 5/5, 4/5 and 6/7, so there is no obvious inversion of rank.

Having made these considerations, the appraiser made the calculations to determine the value of Castle N, for this purpose he presented the following table**:**

Table 17: Results of the Unit Value of Castle N based on the Valuation Ratio (with Castle 6)

| Castle | Unit Value [UM/$m^2$] | Vvg [adim] | Valuation Ratio [UM/$m^2$] | Vvg N [adim] | Unit Value N [UM/$m^2$] |
|---|---|---|---|---|---|
| Castle 1 | 1,650.00 | 0.1676 | 13,882.92 | 0.0949 | 1,317.92 |
| Castle 2 | 1,200.00 | 0.0323 | | | |
| Castle 3 | 1,575.00 | 0.1133 | | | |
| Castle 4 | 2,040.00 | 0.1436 | | | |
| Castle 5 | 3,100.00 | 0.3011 | | | |
| Castle 6 | 3,000.00 | 0.1472 | | | |
| Total | 12,565.00 | 0.9051 | | 1.0000 | |

From this table, the unit value of the assessable property of 1,317.92 UM/$m^2$ is extracted, therefore, the total value of the property is calculated as follows:

Table 18: Total value of Castle N (considering Castle 6)

| Castle | Construction area [$m^2$] | Unit Value* [UM/$m^2$] | Total Value [UM] |
|---|---|---|---|
| Castle N | 1,500.00 | 1,320.00 | 1,980,000.00 |

*rounded

In order to visualize the overall results, the appraiser presented a summary of the calculations made in this phase of the investigation:

[16] It is recommended in these cases to use the ideal Absolute/Synthesis Mode to mitigate rank inversion in AHP.

Table 19: Comparative results of the unit and total values for each case

| Case | Unit Value [UM/m$^2$] | Total Value [UM] |
|---|---|---|
| Original Case 6A | 1,150.00 | 1,725,000.00 |
| Caso 5A (-2) | 970.00 | 1,455,000.00 |
| Caso 5A (-4) | 1,200.00 | 1,800,000.00 |
| Caso7A (+6) | 1,320.00 | 1,980,000.00 |
| Mean | 1,160.00 | 1,740,000.00 |
| Standard Deviation | 145.37 | 218,059.62 |
| Coefficient of Variation | 12.53% | 12.53% |

As can be seen, the average total value of the cases analyzed is 1,740,000.00 UM, with a value range between 1,455,000.00 and 1,980,000.00 UM, with a percentage difference between the highest value and the lowest value, based on the lowest, of +36.08 %.

Now that the appraiser has obtained the results, he must present them to the King to base his decision on the amount to be offered to the nobleman for Castle N. Once again, he went to the Royal Castle to talk to His Highness and present the results of the "task" entrusted to him. When the King saw the results, he wondered why there was so much variation between the highest value and the lowest value, and asked what could be the causes of this "imprecision".
There being no satisfactory response from the appraiser, they reached an agreement to analyze the changes in the determined values of the comparable castles. The task would be as follows: For each case analyzed, determine the value of each castle using the valuation ratio, but including the determined value of Castle N, i.e., assume it as valid, from there draw the castle to be appraised with the procedure. This operation must be repeated for each of the castles, obtaining a value that must be compared with the price offered.
In addition, a simple fit test called Non-Ratio Reciprocity Test (NRRT) should be performed[17], which consists of calculating the percentage difference between the estimated value and the offered price of the comparable properties, through the use of the Valuation Ratio, using the following expression:

$$\boldsymbol{NRRT = \frac{Estimated\ value\ -\ offered\ price}{Offered\ price}} \qquad (3)$$

This is how the appraiser decided to perform the calculations that allowed him to meet the real requirements. For the first case, the Original Case of the six alternatives, the calculation of the value of Castle 1 is presented:

Table 20: Value of Castle 1 using the Valuation Ratio (Original Case)

| Castle | Unit value [UM] | Vvg [adim] | Valuation ratio [UM/m$^2$] | Vvg 1 [adim] | Unit Value 1 [UM/m$^2$] | Unit price [UM/m$^2$] | NRRT |
|---|---|---|---|---|---|---|---|
| Castle 2 | 1,200.00 | 0.0389 | 11,063.26 | 0.1814 | 2,007.09 | 1,650.00 | 22% |
| Castle 3 | 1,575.00 | 0.1322 | | | | | |
| Castle 4 | 2,040.00 | 0.1966 | | | | | |
| Castle 5 | 3,100.00 | 0.3443 | | | | | |
| Castle N | 1,141.17 | 0.1066 | | | | | |
| Totals | 9,056.17 | 0.8186 | | 1.0000 | | | |

[17] Camacaro (2022) Test proposed by the author of this article that can be seen in the following video "Valuation Ratio + Control Metrics" by accessing through this link: https://youtu.be/50Kf64WAv9s

As can be seen, the calculated value is 2,007.09 UM/$m^2$ while the offer price is 1,650.00 UM/$m^2$, generating an increase of 22% compared to the initial price. All calculations were made in this way for all cases and all castles, generating the following table:

Table 21: Comparison of the results of the unit, total and NRRT values for each case

| | | Original Case | | Original Case (-A2) | | Original Case (-A4) | | Original Case (+1A) | |
|---|---|---|---|---|---|---|---|---|---|
| Castle | Offers | Unit Value [UM/$m^2$] | NRRT [adim] | Unit Value [UM/$m^2$] | NRRT [adim] | Unit Value [UM/$m^2$] | NRRT [adim] | Unit Value [UM/$m^2$] | NRRT [adim] |
| 1 | 1,650.00 | 2,007.09 | 22% | 1,804.97 | 9% | 2,193.26 | 33% | 2,326.96 | 41.03% |
| 2 | 1,200.00 | 384.63 | -68% | NA | NA | 382.98 | -68% | 422.99 | -64.75% |
| 3 | 1,575.00 | 1,390.80 | -12% | 1,164.02 | -26% | 1,341.61 | -15% | 1,572.53 | -0.16% |
| 4 | 2,040.00 | 2,120.60 | 4% | 1,885.37 | -8% | NA | NA | 1,764.75 | -13.49% |
| 5 | 3,100.00 | 3,994.44 | 29% | 3,663.18 | 18% | 4,063.90 | 31% | 5,102.38 | 64.59% |
| 6 | 3,000.00 | NA | NA | NA | NA | NA | NA | 1,878.27 | -37.39% |
| N | NA | 1,141.17 | 0% | 969.40 | 0,00% | 1,192.61 | 0% | 1,192.05 | 0,00% |

The appraiser, surprised by the results, decided to meet with his colleagues before presenting them to the King. In an interesting discussion, the following conclusions were reached:

a) In all cases, all the castle values were very different, especially Castle 2, which with a price of 1,200.00 UM/$m^2$, the values are located between 384.63 and 422.99 UM/m2 with a maximum NRRT of -68%.

b) For Castle N, it is observed that it went from a minimum of 969.40 UM/m2 to a maximum of 1,192.61 UM/$m^2$, a 23% variation with respect to the lowest value.

c) When a castle is removed, positive and negative NRRTs are observed, so that there is an underestimation and overestimation of the initial prices.

d) For the case of the six alternatives, there are radical changes in the estimated values, reflected in the high percentages of the NRRT.

e) The valuation ratio is different in all cases.

Not very convinced by these results, but as there was a rush to make the decision, he decided to visit the Sovereign again. Once the report was presented, the King made an even more conservative reading of the section of the report where reference was made to the procedure used: Multicriteria Method + Valuation Ratio in the appraisal. In this regard, His Highness, with the serenity that characterized him, knowing that it was not his specialty, considered based on the results that there is no reliability, there is no consistency and much less reciprocity in the prices used for the calculations and the values determined.

He continued by saying that if a scenario is proposed using the same data, hiring several expert appraisers and using the same method, it is very likely that different results will be obtained, this, due to the subjectivity present in the pairwise comparison of the criteria and the alternatives by the criteria to form the elements of the matrices on which the AHP is based, which to his "Real, Fair, Know and Understand", even if a mathematical artifice is used, it would not be possible to obtain market values.

Before continuing, the King commented on his *mea culpa* in this appraisal work, since he imposed conditions on the appraiser such as: the size of the sample, the characteristics of the properties, as well as the delivery time, which put the appraiser against the "shields" of the Royal Castle, who in order to please the will of the Lord of the Kingdom did not oppose the conditions of the assignment.

As in every kingdom, kings always have their wise collaborators, so the King asked one of them to briefly explain the AHP, another to talk about the econometric models and the royal valuation standards.

He wanted to get a wise answer on the problem of the inaccuracy of the results. Regarding the query on the use of AHP, the wise Analyst explained the following to him:

«We are in the presence of two very different procedures with different objectives. In the first, an attempt is made to solve a Selection problem -Pα[18], that is, choosing a subset of alternatives as restricted as possible, taking into account the final choice of a single action. This set will contain the "best actions" or the "satisfactory" actions. The result sought is, therefore, a choice based on multiple criteria.

This is effectively achieved by using a multi-criteria decision, which is the set of approaches, methods, models, techniques and tools aimed at improving the overall quality of the decision processes followed by individuals and systems, that is, to improve the effectiveness, efficiency and efficacy of the decision processes, and to increase knowledge of these (added value of knowledge)[19].

In this case, one of the multi-criteria decision analysis (MCDA) methods was used, the Analytic Hierarchy Process (AHP) which considers three main steps: first, structuring; second, measurement; and third, synthesis. In this particular case, the original AHP was used, that is, a relative measurement/normalized synthesis was made which allowed obtaining the priority vectors of the alternatives by the criteria, of the criteria and the global to determine a ranking of the alternatives.

Among the characteristics used are the quality of the environment where the castle is located, its quality and state of conservation. Pairwise comparison judgments were made and the elements of the matrices were completed with the values established in the Saaty fundamental scale, and through a series of mathematical calculations a global vector of the alternatives was obtained to obtain an order that allows the selection of the alternative that has the highest value among the elements of the vector.

The mathematically determined values, which are dimensionless, are directly linked to the measurement of the criteria and alternatives at the hierarchical levels considered, therefore, they are not binding for the exchange of information with other measurements or indicators of the real estate market, which has a very different behavior, and which must be analyzed separately.

To check the non-linkage let's do an exercise considering the judgments of the King, the Royal Valuator, the Analytic and the also invited, the wise Regressor, but this time let's consider the characteristics: strength, security and access to the castles. "Remember that if there is no unanimity in the comparisons, we will use the geometric mean of the individual judgments to complete the matrices," exclaimed the sage. This time there was an agreement, the following reciprocal matrices of pairwise comparisons of the criteria and of the alternatives by criteria were generated:

Table 22: Reciprocal matrix of criteria and priority vector

| Criteria | Strength | Security | Access | Vector normalized priority |
|---|---|---|---|---|
| Strength | 1 | 1/4 | 1/4 | 0.10852 |
| Security | 4 | 1 | 2 | 0.54693 |
| Access | 4 | 1/2 | 1 | 0.34454 |

Consistency ratio= 5,16%

[18] Gomes, C. F. S., & Costa, H.G. (2015). *Application of multi-criteria methods to the problem of choosing electronic credit card payment models. Production, 25* (1)

[19] Moreno Jiménez (2001) "The Analytic Hierarchy Process (AHP). Fundamentals, Methodology and Applications"

Table 23: Reciprocal matrix of alternatives by strength criterion and normalized priority vector [20]

| Strength | Castle 1 | Castle 2 | Castle 3 | Castle 4 | Castle 5 | Castle N | Normal priority vector |
|---|---|---|---|---|---|---|---|
| Castle 1 | 1 | 1/5 | 1/3 | 1/2 | 1/3 | 1/5 | 0.0498 |
| Castle 2 | 5 | 1 | 3 | 4 | 1/3 | 2 | 0.2331 |
| Castle 3 | 3 | 1/3 | 1 | 4 | 1/4 | 1/2 | 0.1156 |
| Castle 4 | 2 | 1/4 | 1/4 | 1 | 1/5 | 1/3 | 0.0578 |
| Castle 5 | 3 | 3 | 4 | 5 | 1 | 3 | 0.3805 |
| Castle N | 5 | 1/2 | 2 | 3 | 1/3 | 1 | 0.1633 |
| | Consistency ratio= | 7.88 % | | | | | |

Table 24: Reciprocal matrix of alternatives by safety criterion and normalized priority vector

| Security | Castle 1 | Castle 2 | Castle 3 | Castle 4 | Castle 5 | Castle N | Normal priority vector |
|---|---|---|---|---|---|---|---|
| Castle 1 | 1 | 7 | 5 | 3 | 1 | 3 | 0.3550 |
| Castle 2 | 1/7 | 1 | 2 | 1/2 | 1/2 | 1/3 | 0.0747 |
| Castle 3 | 1/5 | 1/2 | 1 | 2 | 1/3 | 1/5 | 0.0715 |
| Castle 4 | 1/3 | 2 | 1/2 | 1 | 1/3 | 1/2 | 0.0864 |
| Castle 5 | 1 | 2 | 3 | 3 | 1 | 1 | 0.2143 |
| Castle N | 1/3 | 3 | 5 | 2 | 1 | 1 | 0.1981 |
| | Consistency ratio= | 7.80 % | | | | | |

Table 25: Reciprocal matrix of the alternatives by access criterion and normalized priority vector

| Access | Castle 1 | Castle 2 | Castle 3 | Castle 4 | Castle 5 | Castle N | Normal priority vector |
|---|---|---|---|---|---|---|---|
| Castle 1 | 1 | 4 | 1/3 | 1/3 | 4 | 2 | 0.1740 |
| Castle 2 | 1/4 | 1 | 1/3 | 1/3 | 1/3 | 1/5 | 0.0501 |
| Castle 3 | 3 | 3 | 1 | 1 | 3 | 1 | 0.2399 |
| Castle 4 | 3 | 3 | 1 | 1 | 5 | 3 | 0.3087 |
| Castle 5 | 1/4 | 3 | 1/3 | 1/5 | 1 | 1/3 | 0.0692 |
| Castle N | 1/2 | 5 | 1 | 1/3 | 3 | 1 | 0.1582 |
| | Consistency ratio= | 9.79 % | | | | | |

Once the priority vectors of the criteria and the alternatives by criteria were obtained, the global vector and the ranking were calculated and are shown in the following table:

Table 26: Results of the global vector and ranking of the alternatives

| Castle | Strength | Security | Access | Vector priority criteria | Normalized global vector | Ranking |
|---|---|---|---|---|---|---|
| Castle 1 | 0.04980 | 0.3550 | 0.1740 | 0.10852 | 0.2595 | 3 |
| Castle 2 | 0.23309 | 0.0747 | 0.0501 | 0.54693 | 0.0834 | 6 |
| Castle 3 | 0.11560 | 0.0715 | 0.2399 | 0.34454 | 0.1343 | 4 |
| Castle 4 | 0.05777 | 0.0864 | 0.3087 | | 0.1599 | 2 |
| Castle 5 | 0.38047 | 0.2143 | 0.0692 | | 0.1823 | 1 |
| Castle N | 0.16328 | 0.1981 | 0.1582 | | 0.1806 | 5 |
| Consistency ratio= | 7.88 % | 7.80 % | 9.79 % | 5.16 % | | |

[20] All results were obtained using Excel spreadsheets and checked with the free Super Decisions program, which can be downloaded from the web site through this link: *https://www.superdecisions.com/*

With the values of the elements for each alternative of the normalized global vector and following the procedure explained above, the unit value of Castle N was determined, which can be seen in the following table:

Table 27: Results of the Unit Value of Castle N based on the Valuation Ratio

| Castle | Unit Value [UM/m$^2$] | Vvg [adim] | Valuation Ratio [UM/m$^2$] | Vvg N [adim] | Unit Value N [UM/m$^2$] |
|---|---|---|---|---|---|
| Castle 1 | 1,650.00 | 0.2595 | 11,672.56 | 0.1806 | 2.107.56 |
| Castle 2 | 1,200.00 | 0.0834 | | | |
| Castle 3 | 1,575.00 | 0.1343 | | | |
| Castle 4 | 2,040.00 | 0.1599 | | | |
| Castle 5 | 3,100.00 | 0.1823 | | | |
| Total | 9,565.00 | 0.8194 | | 1.0000 | |

We now proceed to calculate the total value of the Castle N:

Table 28: Total value of Castle N

| Castle | Construction area [m$^2$] | Unit Value* [UM/m$^2$] | Total Value [UM] |
|---|---|---|---|
| Castle N | 1,500.00 | 2,100.00 | 3,150,000.00 |

*rounded

Now, a comparison is made with the results obtained, which are shown in the following table:

Table 29: Comparative table of the total value of the Castle N

| Case | Unit Value [UM/m$^2$] | Total Value [UM] | Percentage difference |
|---|---|---|---|
| 1 | 1,150.00 | 1,725,000.00 | |
| 2 | 2,100.00 | 3,150,000.00 | 82.61 % |

As can be evidenced, admired Highness, there is a significant difference of 82.61% between the two cases, which is sufficient evidence to conclude that the results of the global vector of alternatives are not binding with market prices. Although the AHP is correctly applied, the mathematical artifice does not guarantee reliable real estate market values. That's all, exclaimed the Analytical sage. I now give way to the wise Regressor to expound his knowledge of the controversial procedure».

The sage Regressor stated the following:

«I agree with the position of the Analytical sage, because scientifically they are two procedures that have very different and independent objectives and results. When the Comparative Market Data Method is used, the real standard[21] indicates that two types of treatment can be performed: factor-based and scientific.

[21] According to the content of the Brazilian standard, item 8.2.4.1 Data processing of ABNT NBR 14653 Property appraisals. Part 2: urban real estate (2011). P. 15.

The first is very subjective and easy to solve, while the second requires advanced statistical knowledge that allows econometric modeling of real estate market prices and the ability to estimate market values with a certain level of confidence. For both cases, the sample size is very important.
In the case analyzed, the sample is of six comparables, then one more was added: Using the formula 3(k+1) contained in the standard in reference, for example, for the Foundation Grade I, as there were three variables analyzed, 3*(3+1) would be needed, that is, 12 data, therefore, even if a treatment not contained in the standard is not being applied, it is obvious that there is a non-compliance.
Likewise, for the selection of the variables, no in-depth research was carried out to allow the appraiser to know if they are statistically significant[22].
The verification exercise developed by the Analytical sage, assuming that his Majesty could have selected other variables, for example, the strength, the security and the access of the castle, which could have been other more quantitative variables, allowed corroborating the non-sustainability of the results of the values of the appraisable real estate.
If you remember, Your Highness, your kingdom approved a Royal Order on the rules for the valuation of real estate that explicitly states the following:

> "In order to use the Comparison Method for the purposes of this Order, the following requirements must be met: a) The existence of a representative market of comparable properties. b) Sufficient data on transactions or offers that allow, in the area in question, the identification of adequate parameters to carry out the homogenization of comparables. c) Sufficient information on at least six transactions or offers of comparables that adequately reflect the current situation of said market. d) The availability of sufficient information on at least six transactions or offers of comparables that adequately reflect the current situation of said market. e) The availability of sufficient information on at least six transactions or offers of comparables that adequately reflect the current situation of said market"[23].

It is obvious that, in the cases analyzed, the minimum number of comparable offers required by the regulations is not met. Likewise, the homogenization of comparables with the criteria, coefficients and/or weightings that are appropriate for the property in question must be complied with, and precisely this procedure is not achieved with the Multicriteria method.
As a conclusion, when applying the AHP with the Valuation Ratio it is not possible to obtain market values because, as the wise Analytic assured, the values of the global vector are not binding with the offered prices of the property, due to the origin and the very nature of the information contained in the decision and estimation elements used in this procedure.
For a greater empirical foundation on the inadvisability of using the AHP+RV in the real estate valuation, I expose to you, with all respect Your Highness, an analogous situation considering econometric models, which must follow these steps:

1. Define an economic function to explain real estate market prices.
2. To structure an econometric model considering the explained and explanatory variables.
3. Characterize the variables involved in the model.

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[22] See Camacaro & Mock (2022) "An econometric model + Analytic Hierarchy Process (AHP) to select the best alternative for a real estate investment. case: apartment in Panama City". https://youtu.be/vrDYJJjtJsc
[23] Order ECO/805/2003, of March 27, 2003, on rules for the valuation of real estate and certain rights for certain financial purposes. Ministry of Economy. Spain.

4. To raise a market sample based on the variables that the researcher assumes are important in explaining real estate prices.
5. Econometric modeling using multiple linear regression models (MLRM).
6. Analyze the results and conclude on the point and interval estimation of the real estate value.

In order to fulfill the first step, the economic function is proposed:

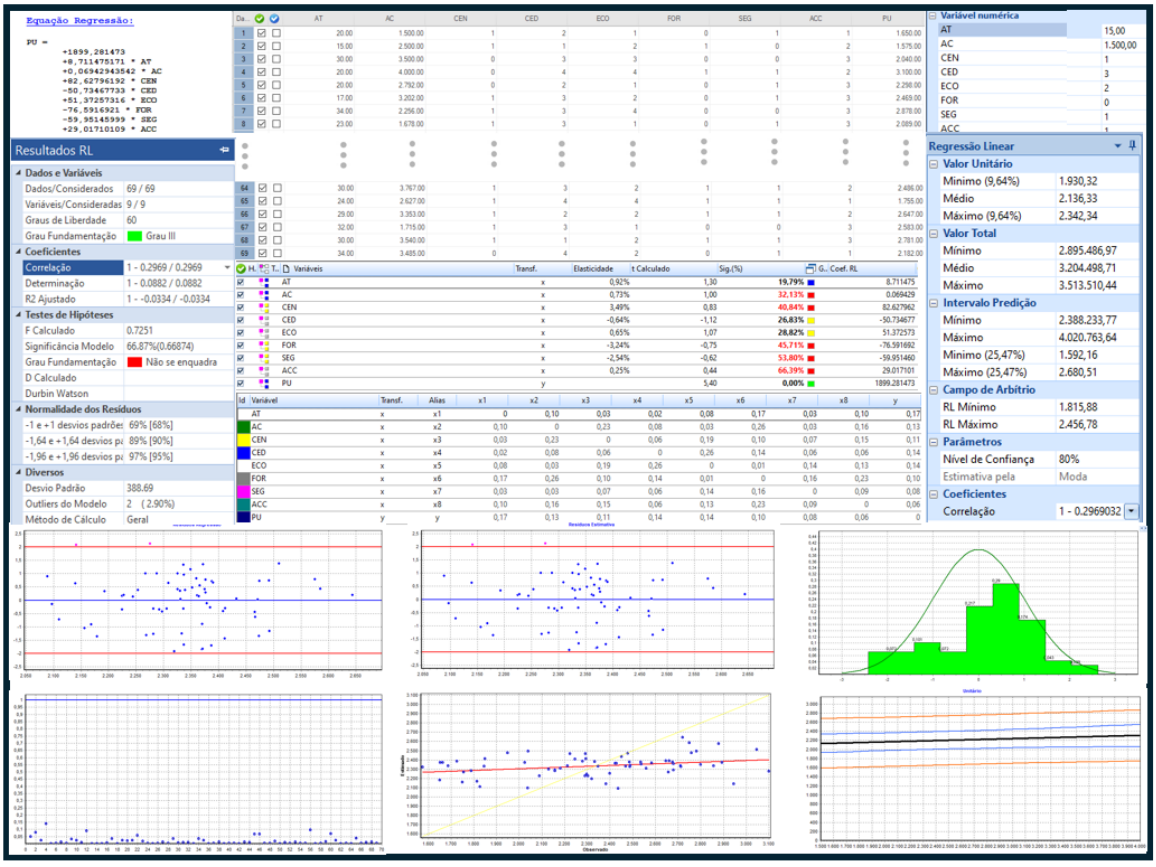
$$PU=(AT,AC,CEN,CED,EC,FOR,SEG,ACC) \quad (4)$$

The regression equation is now posed:

$$Y_i = \beta_0 + \beta_1 AT_i + \beta_2 AC_i + \beta_3 CEN_i + \beta_4 CED_i + \beta_5 EC_i + \beta_6 FOR_i + \beta_7 SEG_i + \beta_8 ACC_i + \varepsilon_i \text{, para } i=1...n \quad (5)$$

For the sample of comparable castles [24] a simulation was performed using the Excel function «*RANDBETWEEN*()» considering the maximum and minimum values of the variables in Table 1. The results of the regression model calculation are as follows (use the SISDEA[25] Program):

Figure 1- Multiple linear regression model results




[24] The sample size was 69, according to numerology, which is the study of the relationship between numbers, letters and patterns, this number is related to humility, humanity and good communication. https://www.oroscopissimi.it/numerologia/numero-69.htm

[25] SISDEA is an advanced software for evaluating urban and rural properties, machinery and equipment, following the guidelines of ABNT NBR 14.653. With the direct comparative method of market data, it offers an accurate and reliable evaluation.Visit: https://pellisistemas.com/software/sisdea-avaliacao-de-imoveis/

From Figure 1, it can be commented that an estimation equation of the additive type was obtained. The rule states that to reach Level III, 6 x (k+1) must be less than the number of data used here is 6 x (8+1) = 54 less than 69, so this condition is met. There were also no imbalance problems in the explanatory variables of the model.

The correlation coefficient (*r*) is equal to 0.30, indicating a weak correlation between the explained variable and the explanatory variables. The coefficient of determination ($R^2$) is equal to 0.0882, indicating that 8.82 % of the variation of the unit values around the mean value is explained by the independent variables included in the model. As for the interpretation of the significance of the regressors of the explanatory variables, all of them have a significance greater than 10%, which does not allow this item to be included in the Degree of substantiation.

The calculated *F* is equal to 0.7251; while the significance level of the model is 66.87 %, which is higher than the significance level α, so the null hypothesis is accepted, therefore the independent variables included in the model are not important for the explanation of the variability of the prices observed in the market..

Regarding the Normality of the errors, comparing the relative frequency of the standardized errors of the sample in the intervals of [-1;1], 65% [-1.64;1.64] 93% and [-1.96;1.96], 98% with probabilities of the standard normal distribution in the same intervals, that is, 68%, 90% and 95%, there is an approximation considering the tolerance for this test.

The standard deviation of the regression model is equal to 388.69 UM/m². The presence of two outliers (2.90 %) is observed, an amount proportionally less than the 5% tolerable for this analysis of normality in the residuals. When reviewing the correlation values, it is evident that there are no high correlations between the variables (less than 0.80), which explains why there is no presence of multicollinearity between the independent variables.

Continuing with the graphical tests on Normality and Homoscedasticity in the errors, in the scatter graph of the values estimated by the model versus the standardized errors (of the estimation and of the regression) we can observe the distribution of the errors between -2 and 2, which corroborates the approximate normality of the errors (the outlier is also observed), while the cloud of points does not present any tendency, which is an indication that they are arranged randomly and it is interpreted that the errors of the model have a homoscedastic behavior, that is, the errors have constant variance.

The histogram of the model errors complements the information on the approximate normality in graphic form by comparing it with the normal distribution.

The presence of influential points in the model can be checked using the Cook Distance graph, in this case it was observed that there are no influential points among the sample data of the regression model.

The adjustment of the model must be checked using the scatter graph of the observed prices on the x-axis and the values estimated by the model on the y-axis, which must contain points close to the bisector of the first quadrant, as can be seen in this graph, the adjustment made by the regression model is checked. When observing the graphs, there is no evidence of maximum and minimum points.

By substituting the values of the variables of the taxable property in the regression equation, which is carried out through the "Project Values" module of the program used, the following was obtained: The point estimate is 2,136.33 UM/$m^2$, and the interval estimate with a confidence level of 80 % means that, if 100 samples of the same size are taken, with the same variables and under the same collection conditions, the average population market value will be contained in 80 of those calculated intervals, and will possibly be contained in the interval between [1,930.32; 2,342.34] UM/$m^2$. Assuming a unit value of 2,200.00 UM/$m^2$, the value of the taxable property is determined:

Table 30: Total value of taxable property (Castle N)

| Castle | Construction area [$m^2$] | Unit Value* [UM/$m^2$] | Total Value [UM] |
|---|---|---|---|
| Castle N | 1,500.00 | 2,200.00 | 3,300,000.00 |

*Rounded

This result does not have a valid empirical basis as shown in the presentation of MRLM calculations, as well as being determined using AHP+RV (3,150,000.00 UM). Despite a percentage difference of +4.76% in the VMRLM in relation to the VAHPRV, it is a mere coincidence of results, both are consistent, proving that the results of the global vector of criteria are not binding with the prices of the real estate market, therefore the AHP method is well applied, the Value Ratio as a mathematical artifice does not guarantee market real estate values.

As a final recommendation, admired King, it is suggested that a market study be carried out to identify quantitative and qualitative variables to characterize a market sample of comparables of medieval buildings, large enough to be able to model market prices that allow us to provide a solution punctual and by interval of market values, in this case, of the castle to buy».

Once the consultation was over, there was nothing left to talk about, the King said he had finished the activity, thanking those who knew him for his important and clarifying information, but he asked the Real Appraiser who remained in the Castillo because he had to plan the investigation to raise the market sample real estate and carry out the entire valuation process to obtain the market value that allows you to make the corresponding offer.

## 4. DISCUSSION

There is no doubt about the effectiveness of AHP in decision making. But the valuers have the obligation to consult the original sources of knowledge, never accept a methodological procedure as valid, there is always a need to prove its empirical foundation because that is where the discovery of information enhances the investigation into the science of valuation.

## 5. CONCLUSION

Based on the ad absurdum argumentative strategy, and with the results of the investigation it was demonstrated that the alternative hypothesis is false, therefore, the results of the general weighting vector of the AHP are not linked to the prices of the real estate market. Therefore, the valuation ratio as a mathematical artifice does not guarantee market real estate values.

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